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ABSTRACT

In this paper I develop an innovative evolutionary framework to integrate Keynes' short-run principle of effective demand and the formation of long-run prices of production in Classical Political Economy. At the intersection of Keynes, Marx, and Kalecki, my evolutionary framework integrates effective demand, functional income distribution, profit rate equalization, technological diffusion, and the gravitation towards prices of production. My approach bridges two gaps at once: the absence of the short-run principle of effective demand in Classical Political Economy; and the absence of technological diffusion, profit rate equalization, and the formation of long-run prices of production in Keynes and Kalecki. To formalize the feedback effects between individual decisions taken at the micro level and the unintended social outcomes at the macro level I develop a simple model using replicator dynamics from evolutionary Game Theory. My approach offers a better understanding of how effective demand determines the rate of exploitation, the equalization of profit rates, and the convergence of market prices towards prices of production.

Key words: Effective Demand, Prices of Production, Marx, Keynes, Kalecki

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1. Introduction

In this paper I develop an innovative evolutionary framework to integrate Keynes' short-run principle of effective demand and the formation of long-run prices of production in Classical Political Economy. At the intersection of Keynes, Marx, and Kalecki, my evolutionary framework integrates effective demand, functional income distribution, profit rate equalization, technological diffusion, and the gravitation of market prices towards prices of production.

The incorporation of Keynes' principle of effective demand into the framework of Classical Political Economy offers a better understanding of how aggregate demand determines the rate of exploitation, the behavior of profit rates, and the formation of prices of production in the long run. My approach therefore bridges two gaps at once. The first gap relates to the absence of the short-run principle of effective demand in Classical Political Economy, most markedly in the canonical works of Smith, Say, Ricardo, and Mill. The usual procedure in Classical Political Economy was to employ Say's Law in either one of its two versions, namely Say's Identity or Say's Equality, hence downgrading aggregate demand to a passive role (Becker and Baumol 1952; Baumol 1977, 1999; Robison 1947; Shoul 1957; Foley 1985). The second gap relates to the absence of technological diffusion, profit rate equalization, and the formation of long-run prices of production in the works of Keynes and Kalecki. Neither Keynes nor Kalecki showed how competition at the micro level would equalize profit rates and converge market prices towards prices of production in the long run.

My framework draws from Marx's insights in chapters IX and XV in *Capital III*, from Keynes' principle of effective demand, and from Kalecki's insights on macroeconomic aggregates. In my approach, aggregate demand determines the amount of value realized. And profit rates equalize across sectors as long as profitability responds negatively to the capital committed to production in each sector. Over-supply in one sector will erode profits in that sector and firms will search for more profitable opportunities elsewhere.

Once effective demand is brought into Classical Political Economy, the distinction between the *source* of value and the determination of the *quantitity* of value becomes crucial. In this paper I take the Marxist stance that *value* is the social form of labor in capitalism, whose quantity is determined by the abstract labor time *socially necessary* to reproduce commodities. As Rubin ([1928]1972) noted, the term "socially necessary" has a double meaning. It means socially necessary in the supply-side sense that the existing technology and cost structures determine what and how firms can produce. But "socially necessarily" also has a demand-side dimension under which value needs social validation to become value in the first place. Hence, the "socially necessary" character of value determination must be understood at the intersection of supply and demand aspects. There is, therefore, no such a thing as non-realized values, for value must be realized to be conceptualized as value. Value that is not realized is not value: "It is only by being exchanged that the products of labour acquire a socially uniform objectivity as values" (Marx [1887]1990, p.166), as the "different kinds of individual labour represented in these particular use values, in fact, become labour in general, and in this way social labour, only by actually being exchanged" (Marx [1859]1989, p. 286).

A central feature of my approach is the distinction between the *ex ante* and the *ex post* rates of exploitation. Marx ([1894]1994, p.352-353) himself introduced this distinction in chapter XV of *Capital III* where he theorized the difference between "immediate exploitation" at the point of production, which is the *source* of value, and "realized exploitation" at the point of exchange, which determines the *quantity* of value. The distinction between the *ex ante* and the *ex post* rates of exploitation is what allows for the integration of Keynes' principle of effective demand, as well as Kalecki's macroeconomic aggregates, into the Marxist framework of accumulation and distribution.

In my framework I additionally introduce competition at the firm level through cost-reducing technological progress. And because the real wage in endogenous to aggregate demand, the Okishio (1961) theorem no longer holds. Okishio (1961) had originally demonstrated that in a Marxist framework with an exogenous real wage, technical change will lead the average profit rate not to fall but in fact to rise. In my evolutionary approach, on the contrary, technical change can reduce profitability over the long run even if it temporarily raises profit rates for the initial adopters.

In this paper I build on the existing scholarship and develop an evolutionary model that closely mimics Marx's original insights on exploitation, accumulation, and technological progress in volume III of *Capital*. I build a simple model using replicator dynamics from evolutionary Game Theory to describe the competitive selection that occurs simultaneously at the micro intra-sector and at the macro inter-sector levels. The replicator dynamics describe an updating process with random interactions in which behaviors with higher payoffs proliferate. It is a useful device to mimic the competitive struggle for survival in natural and social environments, for it models the process of equilibration by tracking the results of individual interactions (Bowles 2006; Gintis 2009; Prado 2006, 2002). The proposed framework formalizes key aspects of the Classical Political Economy approach to accumulation and profitability in an adaptive system, in which agents control their actions but not the aggregate consequences of their individual decisions. Micro decisions produce unintended macro outcomes that then feed back again into micro decisions.

My framework brings together the principle of effective demand from Keynes, the theory of value from Classical Political Economy, and the evolutionary approach to modelling from Game Theory. It therefore allows us to integrate Keynes', Marx', and Kalecki's insights on profit equalization, prices of production, and the role of aggregate demand in the realization of value and surplus value. By incorporating Keynes' principle of effective demand into the Classical Political Economy theory of value and distribution, I am able to offer a better understanding of how aggregate demand determines the behavior of profit rates and of prices of production in the longer run.

2. Effective Demand and Say's Law

The principle of effective demand remains a hotly debated concept even within the post-Keynesian tradition. In this paper I follow Chick (1983), Hayes (2019; 2007), Hartwig (2007), Allain (2009), and

Casarosa (1981) in their understanding of effective demand as the firms' effective commitment to production. Given the technology and cost structure, effective demand refers to how expected profitability determines supply and employment decisions at the *beginning* of the production period. Hence, this *ex ante* commitment to produce is not identical to the *ex post* aggregate expenditures with consumption and investment. Effective demand is the firms' profit-maximizing expected proceeds, an *ex ante* concept relating to expectations but revised in line with *ex post* realized incomes. Even though aggregate demand and aggregate expenditure are not identical, they can be equal when *ex ante* expectations are fulfilled *ex post*, but not otherwise. Therefore, "effective demand is an unfortunate term, for it really refers to the output that will be supplied; in general there is no assurance that it will also be demanded" (Chick 1983, p.65).

In Kalecki's work effective demand is featured but is not identical to that of Keynes. Assuming workers do not save, the capitalists' aggregate expenditures with investment and consumption goods determine the level of aggregate profits. Workers spend what they get, while capitalists get what they spend. Real aggregate gross profit is determined entirely at the macro level, while income and output are set in the interaction between the micro and macro levels (Kriesler 1989). Kalecki derives his theory of effective demand from the national account identities and from the fact that capitalists decide their expenditures but not their incomes. As in Keynes, investment is autonomous from savings, and changes in income ensure that savings accommodate to the level of investment.

In Marxist theory the principle of effective demand means that the *expected* profit rate determines the firms' constant and variable capitals advanced at the *beginning* of the production period, as well as the firms' *supply* of commodities in the current production period. Market prices can change during the production period, but output changes only in the transition from one production period to the next. The beginning-of-period expenditures, which comprise the firms' effective commitment to production, will then realize the values created at the *end of the previous* production period. Because the economy is structured as a chain of production periods (or circuits of capital as Marx put it), the *ex ante* aggregate demand at the beginning of a production period is also at the same time the expenditure that realizes *ex post* the values created in the preceding production period.

In the works of Smith, Say, and Ricardo – on the contrary – growth, cycles, and recessions were not demand-driven but actually supply-driven phenomena. With notable exceptions such as Marx and Malthus, Say's Law was a key element at the core of Classical Political Economy. Ricardo often claimed in many of his writings that cycles are not caused by aggregate demand deficiency but in fact by miscalculations about what to produce and in what proportions. The crucial issue was therefore not a lack of aggregate demand but a temporary mismatch between the composition of aggregate demand and aggregate supply, solved through the continuous movement of capitals across sectors. Say and Ricardo attributed economic crisis not to oversupply but in fact to underproduction (Béraud and Numa 2018; Kates 1997; Becker and Baumol 1952; Baumol 1977, 1999; Vianello 1989).

Becker and Baumol (1952) and Baumol (1977; 1999) were the first to notice the ambiguity of the term 'Say's Law' in Classical Political Economy. This ambiguity is present in Say's own work and further reproduced by James Mill, David Ricardo, Marx, Keynes, Oskar Lange, and also by Kalecki. Becker and Baumol pointed out that Say's Law actually has two versions, one stronger (Say's Identity) and one weaker (Say's Equality). They proposed the following definitions:

(i) *Walras' Law*: Total demand (including the demand for money) equals total supply (including the supply of money). This is just a definition; no direction of causality is implied between supply and demand.

(ii) *Say's Identity (stronger version)*: Total supply automatically becomes, and is identical to, total demand. This happens because no one ever wants to hold cash, so all sales incomes are immediately spent on other goods and services. The demand for money does not affect aggregate demand, supply, or income. Money is just a veil. Recessions and cycles can occur but are entirely supply-driven.

(iii) *Say's Equality (weaker version)*: Demand is supply-led and the equilibrium between aggregate demand and aggregate supply is stable, such that deviations from it are possible but self-correcting. Because of supply-side issues such as coordination problems and miscalculations, recessions and cycles are possible though brief. Money can be used as a store of value, and as a medium of exchange the supply of money is determined endogenously.

Say's Law in either of its two versions and Keynes' principle of effective demand determine in very different ways the direction of causality between the core elements of Marxist theory (Trigg 2006). An example of this is the direction of causality between aggregate demand and the profit rate. A substantial branch of the Marxist tradition would assign the profit rate as the cause and demand as the effect, which amounts to deploying Say's Law and making the economy supply-led. Discussions of the tendency of the profit rate to fall feature this type of reasoning. For underconsumptionists, on the contrary, the profit rate is the effect and aggregate demand is the cause, in which case the profit rate becomes itself endogenous to demand.

Because of his death in 1883, Marx left unfinished the drafts of the second and third volumes of *Capital*. Engels later edited and published the manuscripts in the 1890s, but the connections between effective demand, exploitation, and accumulation were left incomplete. Since the advent of Keynesian and Kaleckian macroeconomics in the 1930s, heterodox scholars have attempted to offer new insights into how the theory of effective demand relates to Marx's theory of value and accumulation (Shaikh 2016, 1989; Foley 1985, 1983; Baran and Sweezy 1968), but a comprehensive model is still needed.

In the next section I discuss how effective demand determines the realization of values and the equalization of profit rates in the formation of prices of production.

3. Exploitation, Profit Rate Equalization, and Production Prices

In chapter IX of *Capital III*, Marx theorized the equalization of profit rates and the formation of long-run prices of production. In chapter XV of the same volume, Marx then considered the difference between the *ex ante* and *ex post* rates of exploitation, explicitly mentioning the role of aggregate expenditures (consumption plus investment) in the realization of exploitation:

> The conditions for *immediate exploitation* and for the *realization of that exploitation* are *not identical*. Not only are they separate in time and space, they are also separate in theory. The former is restricted only by the society's productive forces, the latter by the proportionality between the different branches of production and by the society's power of consumption. And this is determined … by the power of consumption within a given framework of antagonistic conditions of distribution […]. It is further restricted by the drive for accumulation, the drive to expand capital and produce surplus-value on a larger scale (Marx [1894]1994, p.352-353 – emphasis added).

In the 1930s, Kalecki built on Marx's insights to claim that the volume of real aggregate gross profit is determined at the macro level by the capitalists' aggregate expenditures. Assuming workers do not save, capitalists cannot realize more surplus value in the aggregate than their own expenditures (Sardoni 2009; 1989). No matter how large the *ex ante* rate of exploitation in the production sphere, the capitalist class can only realize an *ex post* rate of exploitation that makes total profits match their own expenditures. Kalecki, however, did not explain how profit rates would equalize across sectors and hence ignored prices of production in his analysis (Jossa 1989; Vianello 1989).

The gravitation of market prices toward prices of production has been the object of rigorous study in the Marxist literature. These studies offer a level of technical detail that is much more precise than the numerical and verbal examples that Marx offered in volumes II and III of *Capital*. In this literature there is an agreement on three main results: (i) profit rates do not always equalize and thus prices of production cannot always function as stable attractors for market prices (Harris 1972; Nikaido 1983, 1985; Flaschel and Semmler 1985; Boggio 1985, 1990; Kubin 1990; Duménil and Lévy 1999, 1995; Prado 2006); (ii) market prices that ensure balanced reproduction across sectors are not necessarily the set of prices that can also ensure profit rate equalization (Cockshott 2017); (iii) free capital mobility and competition among capitals lead to the equalization of profit rates, while free labor mobility and competition among workers lead to the equalization of the rates of exploitation (Foley 2018; Cogliano 2011).

On the empirical side of the literature (Scharfenaker and Foley 2017; Scharfenaker and Semieniuk 2017; Fröhlich 2013; Farjoun and Machover 1983) there has been a growing consensus that profitability converges not to a single uniform profit rate but actually to a statistical equilibrium distribution of profit rates with a tent shape around a single peak. Shaikh (2016) shows, however, that profit rates on *new* investment projects (what Keynes labeled the 'marginal efficiency of capital') tend to equalize over time. Scharfenaker and Foley (2017), in particular, developed a model based on thermodynamics and quantal responses to explain the tent-shaped distribution of profit rates across firms. My evolutionary approach does not explain the statistical distribution of profit rates but it does provide a theoretical framework in which effective demand realizes the values created in production and determines the behavior of profit rates in the long run.

Building on top of the existing literature, in the next sections I develop my evolutionary model of accumulation and competition amid technological progress. First, I formalize the macro inter-sector competition through which the aggregate and growing monetary capital of an economy is continuously redistributed between two sectors: sector I produces the means of production and sector II produces the final consumption good. The continuous redirection of monetary capital between sectors takes place according to profit rate differentials. Second, I formalize the micro intra-sector competition in which individual firms within each sector compete against each other via cost-reducing technical change. Innovations are gradually adopted based on profit rate differentials within sectors. I then close the model using the principle of effective demand and make exploitation, profit, and growth all dependent upon the level of aggregate demand. Profit rates equalize and market prices converge toward production prices. At the end I present computer simulations, an analysis of the evolutionary stability of the long-run equilibria, and an explanation of the conditions under which market prices converge to prices of production.

4. Macro Inter-Sector Competition

The economy-wide circuit of capital, which starts and ends with capital in the form of money, and which represents the production period during time t can be summarized through the aggregation in (1). Variables with primes ($'$) are *ex post* (after production has taken place) while variables without primes are *ex ante* (before production takes place):

$$M_t - C_t \begin{cases} LP \\ MP \end{cases} \ldots P \ldots\ C_t' -\ M_t' \tag{1}$$

An initial amount of monetary capital M_t purchases two types of commodities as inputs C_t: labor power (LP) and means of production (MP). During the subsequent production phase ($\ldots P \ldots$) labor power creates more value than its own. The difference between the value that labor power creates and the value of labor power itself is the surplus value. The total value of the gross output C_t' contains the new value added created by productive workers plus the pre-existing value transferred from the means of production. The gross output exchanges for a sum of money represented by the aggregate gross expenditures M_t'. The extra value that workers create and for which they receive no compensation is the basis for the gross profits $\Delta M_t = M_t' - M_t$ in the system.

The economy comprises two sectors, each producing a single type of output using both labor power and means of production. Sector I supplies a homogenous type of means of production. Sector II supplies a homogenous type of final consumption good. Economic events take place temporally, therefore the overlap of any two consecutive circuits of capital can be represented as follows:

$$M_t - C_t \begin{cases} LP \\ MP \end{cases} \ldots P \ldots \; C_t' - \; M_t'$$

$$M_{t+1} - C_{t+1} \begin{cases} LP \\ MP \end{cases} \ldots P \ldots \; C_{t+1}' - \; M_{t+1}' \tag{2}$$

The new circuit formally repeats the preceding one. The crucial causal relation is then that between the total value realized M_t' at the end of the first circuit and the total monetary capital M_{t+1} advanced at the beginning of the following circuit. Because of its supply-led principle, Say's Law in any of its versions would mean that causality runs from M_t to M_t' and then to M_{t+1}. The principle of effective demand, on the contrary, implies that the direction of causality actually runs from the *ex ante* demand M_{t+1} at the *beginning of the new production period* to the realization of the total value M_t' at the *end of the previous production period*.

There is no fixed capital in this economy, so non-labor inputs are circulating capital only. The means of production that enter as inputs in sectors I and II are the previous output of sector I in the preceding production period. Technology is represented by a linear production structure with fixed coefficients and constant returns to scale. Using a_{ji} to indicate the quantity of input *j* per unit of output i, the matrix of input-output coefficients is:

$$A = [a_{ji}] = \begin{bmatrix} a_{11} & a_{12} \\ 0 & 0 \end{bmatrix} \quad \text{with} \quad 0 \leq a_{ji} < 1 \tag{3}$$

Using l_i to indicate the quantity of labor hours per unit of output in sector i, $r_{i,t}$ to indicate the within-sector profit rate per unit of output, p_i to indicate the market price per unit of output, and w to indicate the money wage per work hour, then per unit of output we have $[p_{j,t-1}a_{ji} + wl_i](1 + r_{i,t}) = p_{i,t}$. For each sector the price system is:

$$
\begin{aligned}
[p_{1,t-1}a_{11} + wl_1](1 + r_{1,t}) &= p_{1,t} \\
[p_{1,t-1}a_{12} + wl_2](1 + r_{2,t}) &= p_{2,t}
\end{aligned}
\tag{4}
$$

The first term inside the brackets on the left-hand side represents constant capital, and the second term represents variable capital or the value of labor power, both in money terms. Their summation $[p_{j,t-1}a_{ji} + wl_i]$ is the unit cost. Competition within each sector then simultaneously determines profit rates and prices.

Although the nominal wage per work hour w is exogenously given by the bargaining power between workers and capitalists, the real wage $\frac{w}{p_{2,t}}$ in terms of quantities of the consumption good produced in sector II is determined endogenously. As in the "New Interpretation" (Foley 2018), values are expressed in money terms and workers get their money wages and spend it as they like, not bound to any real wage specified in terms of a bundle of goods. Labor supply and credit are assumed not to be binding constraints on growth.

In each sector there is a collection of several firms and each of them can switch between sectors depending on the average profitability $\bar{r}_{i,t}$. Capitalists commit their capitals to where they expect to profit the most. But once firms flow into a sector aiming at the prevailing $\bar{r}_{i,t}$ they will immediately and unintentionally alter this average profitability. Supposing a very large collection of firms in the economy, we can normalize the total number of firms to unity and then consider only the evolution of population shares, with $f_{1,t}$ representing the fraction committed to sector I and $f_{2,t}$ the fraction committed to sector II:

$$
M_t = f_{1,t}M_t + f_{2,t}M_t = M_{1,t} + M_{2,t} \quad \text{with} \quad f_{1,t} + f_{2,t} = 1 \tag{5}
$$

Outputs $x_{i,t}$ supplied by each sector are the sectoral monetary capitals advanced divided by the respective unit costs. Sectoral supply expands when more monetary capital is advanced in the sector at the beginning of the production period, and it contracts when capitalists withdraw their initial expenditures:

$$x_{i,t} = \frac{f_{i,t} M_t}{\left[p_{1,t-1} a_{1i} + w l_i\right]} = \frac{M_{i,t}}{\left[p_{1,t-1} a_{1i} + w l_i\right]} \tag{6}$$

Within each sector, $M'_{i,t}$ indicates the end-of-period gross expenditures or the valorized monetary capitals that comprise the original monetary capitals $M_{i,t}$ advanced plus the surplus value realized. Market prices $p_{i,t}$ are the end-of-period expenditures divided by quantities supplied:

$$p_{i,t} = \frac{M'_{i,t}}{x_{i,t}} = \frac{M_{i,t}\left(1 + \bar{r}_{i,t}\right)}{x_{i,t}} = \frac{f_{i,t}\, M_t\left(1 + \bar{r}_{i,t}\right)}{x_{i,t}} \tag{7}$$

The monetary capital $M_{i,t}$ committed to sector i at the beginning of the production period in time t is valorized on average to $\left(1 + \bar{r}_{i,t}\right)$ after the output is sold. The fraction $\left(1 + \bar{r}_{i,t}\right)$ includes the replication of the money initially spent plus average profits. Hence, the valorized capital in each sector is $M'_{i,t} = M_{i,t}\left(1 + \bar{r}_{i,t}\right) = f_{i,t}\, M_t\left(1 + \bar{r}_{i,t}\right)$. Using $\widetilde{r_t}$ to indicate the economy-wide weighted average profit rate, such that $(1 + \widetilde{r_t}) = \sum_i f_{i,t}\left(1 + \bar{r}_{i,t}\right)$, the aggregate valorized capital for the entire economy is:

$$M'_t = M_t\,(1 + \widetilde{r_t}) = \sum_i M_{i,t}\left(1 + \bar{r}_{i,t}\right) = \sum_i f_{i,t}\, M_t\left(1 + \bar{r}_{i,t}\right) \tag{8}$$

The shares of the total monetary capital advanced at the beginning of period $t + 1$ change according to the average profitability obtained in period t in each sector:

$$f_{i,t+1} = \frac{M'_{i,t}}{M'_t} = \frac{f_{i,t}\, M_t\left(1 + \bar{r}_{i,t}\right)}{M_t\,(1 + \widetilde{r_t})} = f_{i,t}\,\frac{\left(1 + \bar{r}_{i,t}\right)}{(1 + \widetilde{r_t})} \tag{9}$$

Rewriting it as $\frac{f_{i,t+1}}{f_{i,t}} = \frac{\left(1+\bar{r}_{i,t}\right)}{(1+\widetilde{r_t})}$, subtracting 1 from both sides and using $\Delta f_{i,t+1} = f_{i,t+1} - f_{i,t}$, we then obtain the replicator equation that formalizes the macro competition between capitalists across sectors:

$$\Delta f_{i,t+1} = \mu\, f_{i,t}\, \left(\frac{1}{1+\widetilde{r}_t}\right) [\bar{r}_{i,t} - \widetilde{r}_t] \tag{10}$$

The profitability gap in relation to the economy-wide average determines how capitalists allocate their monetary capitals across sectors. The ad hoc coefficient $\mu \in (0,1]$ indicates that only a fraction of the capitalists in each sector will in fact shift their capital to a different activity that is currently benefitting from higher returns. The complementary fraction $(1-\mu)$ of the firms cannot update their behavior even when return differentials are an incentive for them to do so.

The individual search for profits creates unintended consequences in both the sector-level and the economy-wide average profit rates. Capitalists make decisions based on profit rates prevailing in each sector, but they end up affecting aggregate profitability through their decentralized individual actions to move their capitals from one sector to another. The effects on the aggregate profit rate then feed back into individual decisions about where to commit the monetary capital in the following period.

The equations so far presented describe the growth of output and the evolutionary adjustments that regulate the shares of monetary capitals over time at the macro level. In the next section I turn to the competition for profits through cost-reducing technical change that characterizes the micro-adjustments within each sector.

5. Micro Intra-Sector Competition

Large collections of firms compete for profits within each sector. Markets are intensely competitive, forcing firms to sell at prevailing market prices. The way to increase individual profit lies therefore with the adoption of new cost-reducing technologies. Innovations are generated exogenously and then adopted conditional on enhancing individual profitability. When an individual firm decides upon the adoption of a new productive structure it does so taking the prevailing market price as given. But the individual adoption of the newer technique changes the sector cost structure, and it therefore unintentionally affects the market price. The new market price then operates as a signal for the remaining firms to also adopt the cost-reducing technique. Each sector will thus display a production structure that is a combination of firms producing with the new technique and firms still producing with the old technique.

The economy has three evolutionary processes taking place concurrently. The first is the evolutionary diffusion of new techniques in the sector producing means of production. The second is the evolutionary diffusion of new techniques in the sector producing final consumption goods. The third is the evolutionary distribution of the growing aggregate monetary capital between sectors. An individual decision to adopt a new technique thus triggers a chain of reactions and feedback effects that no individual capitalist can anticipate. Externalities exist in this economy given that firms do not fully internalize the social consequences of their individual actions.

The prevailing technique of production is represented in the set of four technical parameters $(a_{11}^{o}, a_{12}^{o}, l_1^{o}, l_2^{o})$. An innovation $(a_{11}^{n}, a_{12}^{n}, l_1^{n}, l_2^{n})$ can imply the use of more of labor power and means of production, less of both inputs, or more of one input and less of the other (superscript o for 'old' and n for 'new' technique). Rearranging the price equations in (4) we get the profit rate per unit produced using the old technology:

$$r_{i,t}^{o} = \frac{p_{i,t}}{\left[p_{1,t-1}a_{1i}^{o} + wl_i^{o}\right]} - 1 \tag{11}$$

Similarly, the profit rate associated with the new technology is:

$$r_{i,t}^{n} = \frac{p_{i,t}}{\left[p_{1,t-1}a_{1i}^{n} + wl_i^{n}\right]} - 1 \tag{12}$$

The evolutionary diffusion of a new technique can then be formalized with the dynamics of replication. The variable $\upsilon_{i,t} \in [0,1]$ indicates the share of firms in sector i that adopt the new technique at time t, while $(1 - \upsilon_{i,t})$ indicates the share that remains with the older technique. Because each sector has a large collection of firms, and assuming that they interact through random pairwise matching, we can use a simple replicator equation for the diffusion of innovations. Normalizing population sizes to unity allows us to work with population shares in each sector as follows:

$$\begin{aligned} \upsilon_{i,t+1} &= \upsilon_{i,t} + \upsilon_{i,t}\left(1-\upsilon_{i,t}\right)[r_{i,t}^{n} - r_{i,t}^{o}] \\ \Delta\upsilon_{i,t+1} &= \upsilon_{i,t}\left(1-\upsilon_{i,t}\right)[r_{i,t}^{n} - r_{i,t}^{o}] \\ \Delta\upsilon_{i,t+1} &= \upsilon_{i,t}[r_{i,t}^{n} - \bar{r}_{i,t}] \end{aligned} \tag{13}$$

The term $\upsilon_{i,t}\left(1-\upsilon_{i,t}\right)$ is the variance of the firms within each sector and the term $\left[r_{i,t}^{n} - r_{i,t}^{o}\right]$ is the differential replication selection, so that the updating process is payoff monotonic. The third line in equation (13) follows from the fact that the average profit rate in each sector is: $\bar{r}_{i,t} = \left(\upsilon_{i,t}\right)\left[r_{i,t}^{n}\right] + \left(1-\upsilon_{i,t}\right)\left[r_{i,t}^{o}\right]$.

Technical change and its evolutionary diffusion imply that older and newer cost structures coexist until the newer technique completely replaces the older one. Given the monetary capital $M_{i,t}$ committed to each sector, the new quantities supplied can be found by dividing the monetary capital advanced by the mixed cost structure:

$$x_{i,t} = \frac{M_{i,t}}{\left(\upsilon_{i,t}\right)\left[p_{1,t-1}a_{1i}^{n} + wl_{i}^{n}\right] + \left(1-\upsilon_{i,t}\right)\left[p_{1,t-1}a_{1i}^{o} + wl_{i}^{o}\right]} \tag{14}$$

The supply equation in (14) thus replaces the supply equation in (6), which only applied to production under a single technology. Average rates of profit in each sector now depend on the prevailing market prices and on the linear combination between older and newer techniques:

$$\bar{r}_{i,t} = \frac{p_{i,t}}{\left(\upsilon_{i,t}\right)\left[p_{1,t-1}a_{1i}^{n} + wl_{i}^{n}\right] + \left(1-\upsilon_{i,t}\right)\left[p_{1,t-1}a_{1i}^{o} + wl_{i}^{o}\right]} - 1 = \frac{\Delta M_{i,t}}{M_{i,t}} \tag{15}$$

As soon as profit rates in each sector change from their previous position they trigger intra-sector competition via the micro replicator dynamic in equation (13) as well as inter-sector competition via the macro replicator dynamic in equation (10). The out-of-equilibrium adjustments and the evolution of the system over time explicitly reflect the interplay of unintended social consequences of uncoordinated individual actions. In the next section I analyze the stationary states that might prevail in the long run.

6. Long-Run Equilibria and Evolutionary Stability

The model becomes more intuitive if we focus on the trajectories of the three replicator equations $(f_{1,t}, v_{1,t}, v_{2,t})$ toward their long-run stationary states. Stationary states are those states at which the replicator reaches a fixed point with no further changes in the replication process $(\Delta f_{1,t} = 0, \Delta v_{1,t} = 0, \Delta v_{2,t} = 0)$. The crucial procedure is to know which strategies are going to prevail asymptotically when $t \rightarrow \infty$.

In an evolutionary game with replicator dynamics we know that the evolutionarily stable strategies prevail over the long run. An *evolutionarily stable strategy* (ESS) is a best response to itself and hence it is a symmetric Nash equilibrium that is also asymptotically stable in its respective replicator equation. *Evolutionary stability* implies both self-correction and asymptotic attractiveness (i.e., it is a stable attractor), hence the system converges over time to a stationary point that is evolutionarily stable (Bowles 2006; Gintis 2009; Elaydi 2005; Scheinerman 2000).

In Table 1 I summarize the stationary states and asymptotic properties of each replicator equation. Note that in the macro inter-sector dynamic, when there is no ESS, the system converges to an interior stable solution f_1^* such that average profit rates are equalized asymptotically across sectors. In this case, profit rates are not just *equal* across sectors but truly *equalized* in the sense that the equality in sector profitability is evolutionarily stable. In the Appendix I provide a formal stability analysis of the stationary states.

[Table 1 about here]

Because the technical coefficients in the input-output matrix are exogenous but not constant, a strategy that was an ESS *before* the technical change might not be an ESS *after* the innovation is introduced. As long as we have exogenous innovations brought into the system, the ESS's themselves will change over time. The closure imposed on the system will determine which strategies are ESS, which long-run equilibrium will prevail, and whether or not the stationary state will be stable. In the next section I use Keynes' principle of effective demand as the model closure.

7. Effective Demand and the Realization of Value

In this section I offer a model closure in which the realization of value and surplus value is endogenous and dependent upon Keynes' principle of effective demand. Once effective demand is brought into the framework the realized rate of exploitation, the profit rates and, hence, the distribution of income between wages and profits become dependent upon the demand side.

Because effective demand determines the amount of surplus value realized and hence the *ex post* rates of exploitation, profitability becomes sensitive to the amount of monetary capital committed to each sector. Profit rates equalize as long as the average profit rate of a sector increases less than the competing profit rate when the firms committing their capital to that sector increase their share in the population, or simply $\frac{\mathrm{d}\bar{r}_{1,t}}{\mathrm{d}f_{1,t}} < \frac{\mathrm{d}\bar{r}_{2,t}}{\mathrm{d}f_{1,t}}$. In the Appendix I show under what parameter values this stability condition is met.

In the circuit of capital that Marx developed the total expenditures on labor power and means of production take place at the *beginning* of each production period. This implies that constant capital and variable capital are both *advanced* before production takes place. The hours worked per unit of output, l_i, then generate the value added that corresponds to the summation of wages and profits. Workers in sector i produce $wl_i(1 + e_{i,t})$ of value added per unit of output but only get back the value of their labor power corresponding to wl_i, leaving the surplus $e_{i,t}wl_i$ to the firms hiring them. The realized rate of exploitation $e_{i,t}$ is endogenous to the level of effective demand.

The wage share in the overall value added is $\frac{V}{V+S} = \frac{1}{1+e}$, in which V is the value of labor power (the total wage bill advanced in the economy), S is realized surplus value or profits, $e = \frac{S}{V}$ is the economy-wide rate of exploitation, and V+S is the flow of value added in the economy. Profits originate from unpaid labor time. Hence:

$$M'_{i,t} = \left[p_{1,t-1}a_{1i} + wl_i(1 + e_{i,t})\right]x_{i,t} \tag{16}$$

$$\Delta M_{i,t} = M'_{i,t} - M_{i,t} = e_{i,t}\, wl_i\, x_{i,t} \tag{17}$$

This particular relation between profitability and exploitation derives from the fact that the price system is such that $p_{i,t} = p_{1,t-1}a_{1\mathrm{i}} + wl_i + e_{i,t}wl_i = \left[p_{1,t-1}a_{1\mathrm{i}} + wl_i\right]\left(1 + r_{i,t}\right)$. Rearranging terms and solving for the profit rate gives us:

$$r_{i,t} = \frac{e_{i,t}}{1 + \left(\frac{p_{1,t-1}}{w}\right)\left(\frac{a_{1i}}{l_i}\right)} \tag{18}$$

Equation (18) is the usual Marxist relation in which the profit rate is the rate of exploitation divided by one plus the organic composition of capital. The organic composition is, in turn, the relative price $\frac{p_{1,t-1}}{w}$ times the technical composition $\frac{a_{1i}}{l_i}$ between constant and variable capital.

Once firms begin to adopt technological innovations, the mixed productive structure requires weighting the surplus value produced by the respective shares of firms employing the newer and older technologies. Equations (19) and (20) replace equations (16) and (17) as soon as a new technique is introduced:

$$M'_{i,t} = \left\{\left(v_{i,t}\right)\left[p_{1,t-1}a^n_{1i} + wl^n_i\left(1 + e_{i,t}\right)\right] + \left(1 - v_{i,t}\right)\left[p_{1,t-1}a^o_{1i} + wl^o_i\left(1 + e_{i,t}\right)\right]\right\}x_{i,t} \tag{19}$$

$$\Delta M_{i,t} = M'_{i,t} - M_{i,t} = \left\{\left(v_{i,t}\right)\left[wl^n_i e_{i,t}\right] + \left(1 - v_{i,t}\right)\left[wl^o_i e_{i,t}\right]\right\}x_{i,t} \tag{20}$$

Increments in the share of firms adopting the new technology $(\upsilon_{i,t})$ can reduce or increase the average profit rate prevailing in a sector. But the final effect on profitability can only be known after the repricing of both the means of production and the final consumption good.

The central issue is the determination of the aggregate demand, the surplus value realized, and the endogenous rates of exploitation. In this closure I opt for the neo-Keynesian autonomous investment function à la Joan Robinson (1962; see also Dutt 2011; 1990; Marglin 1984) which assumes that firms operate at full capacity utilization and that the amount of monetary capital committed to the investment good sector is a function of past profitability. The parameters γ_i indicate the sensitivity of *ex ante* investment demand to the observed profit rates in each sector, and the autonomous component is simply the investment carried out in the previous period $(M'_{1,t-1})$. Given that there are firms operating with the newer and older technologies simultaneously in each sector, we have that:

$$\begin{aligned} M_{1,t+1} = M'_{1,t} = M'_{1,t-1} + \gamma_1\{(\upsilon_{1,t})[r^n_{1,t-1}] + (1-\upsilon_{1,t})[r^0_{1,t-1}]\}\, M_{1,t-1} \\ + \gamma_2\{(\upsilon_{2,t})[r^n_{2,t-1}] + (1-\upsilon_{2,t})[r^0_{2,t-1}]\}\, M_{2,t-1} \end{aligned} \tag{21}$$

The monetary capital $M_{1,t+1}$ effectively committed to production in sector I at the beginning of period $t+1$ reflects the capitalists' expected profitability in that sector. Expected profitability is based on the realized profit rate in the previous period. Keynes' principle of effective demand implies that causality runs from $M_{1,t+1}$ at the beginning of production period in $t+1$ to $M'_{1,t}$ at the end of production period in t. The monetary capital $M_{1,t+1}$ advanced is the *ex ante* demand at the beginning of period $t+1$, and as such it simultaneously comprises the expenditure $M'_{1,t}$ necessary to realize the value produced in sector I at the end of the previous production period t.

In sector II, likewise, the monetary capital $M_{2,t+1}$ effectively committed to production at the beginning of period $t+1$ reflects the capitalists' expected profitability for that sector. Supposing that workers do not save and that there is no consumption credit, the total expenditure $M'_{2,t}$ with the consumption goods produced in sector II is simply the total wage bill in the economy. At the beginning of period $t+1$, capitalists commit to sector II an amount of monetary capital proportional to the aggregate consumption of out wages realized in the previous production period t. Given that the wage bills in each sector must be weighted by the shares of firms using the old and the new technologies, we have that:

$$M_{2,t+1} = M'_{2,t} = \{(v_{1,t})[wl_1^n] + (1 - v_{1,t})[wl_1^0]\}\, x_{1,t} + \{(v_{2,t})[wl_2^n] + (1 - v_{2,t})[wl_2^0]\}\, x_{2,t} \tag{22}$$

Therefore, effective demand at the beginning of period $t+1$ is $M_{t+1} = M_{1,t+1} + M_{2,t+1} = M'_{1,t} + M'_{2,t}$, in which the second equality follows directly from equations (5) and (9). The endogenous rates of exploitation $e_{i,t}$ within each sector are the sector surplus values realized over the nominal wage bill advanced:

$$e_{i,t} = \frac{M_{i,t+1} - M_{i,t}}{\{(v_{i,t})[wl_i^n] + (1 - v_{i,t})[wl_i^0]\}\, x_{i,t}} = \frac{M'_{i,t} - M_{i,t}}{\{(v_{i,t})[wl_i^n] + (1 - v_{i,t})[wl_i^0]\}\, x_{i,t}} \tag{23}$$

The rates of exploitation in each sector depend directly on the level of aggregate demand from equations (21) and (22). In *qualitative* terms, profits originate from surplus value. The principle of effective demand then also implies that in *quantitative* terms the determination runs from profits (or realized surplus value) to realized exploitation. Even though profits originate *qualitatively* from surplus value at the point of production, under the principle of effective demand the amount of profits is the *quantity* of surplus value realized at the point of exchange.

In some neo-Kaleckian models (as in Dutt 1990, 1984; Marglin 1984; Badhuri and Marglin 1990) the markup is exogenous and prices are fixed per unit of output; thus income distribution between wages and profits is exogenous. Effective demand determines the level of aggregate output and income via quantity adjustments. But this is not the case in Marx's circuit of capital because the beginning-of-period aggregate expenditures on wages and means of production are *advanced* capital, and are therefore already set at their nominal levels at the start of each production period.

8. Model Simulation

To simulate the model it is necessary to fix parameters and initial conditions. As I show in the Appendix, the long-run stationary state is dependent on the parameter values but independent from the arbitrary initial conditions.

In this example the initial technical coefficients are set to $(a_{11}^o, a_{12}^o, l_1^o, l_2^o)$ = (0.2, 0.1, 0.7, 0.7) representing the old technology. The nominal wage w is set to 10 dollars per work hour and only μ=20% of the firms migrate to another sector in each period according to inter-sector average profitability differentials. The initial aggregate monetary capital $M_{t=1}$ is set to 100 dollars, and the initial distribution is set at 60% to sector I ($f_{1,t=1} = 0.6$) and 40% to sector II ($f_{2,t=1} = 0.4$). The means of production are initially priced at 50 dollars per unit ($p_{1,t=0} = 50$). For the investment function I set $\gamma_1 = \gamma_2 = 0.5$, and investment demand begins at 50 dollars ($M'_{1,t=1} = 50$).

The model is set to run for 400 production periods. For the first 49 rounds the trajectories evolve without technical change. At period $t = 50$ I introduce an innovation in sector II that increases labor productivity by 100% while increasing the use of machines by 100% per unit of output, hence $(a_{11}^o, a_{12}^n, l_1^o, l_2^n)$ = (0.2, 0.2, 0.7, 0.35). This machine-intensive labor-saving innovation generates a strong increase in the technical composition of capital in the sector producing the consumption good. At time $t = 100$ I introduce an innovation in sector I that increases labor productivity by 150% and the use of machines by 100% per unit of output such that $(a_{11}^n, a_{12}^n, l_1^n, l_2^n)$ = (0.4, 0.2, 0.28, 0.35). This innovation implies a strong machine-intensive labor-saving technical change in the sector producing the means of production.

[Figure 1 about here]

In Figure 1 I report simulation results for key variables. Panel (a) shows the equalization of profit rates over time, such that the economy-wide average profit rate is a stable attractor to the sector profit rates. Panel (b) shows the movement of firms across sectors in search of higher returns. Panels (c) and (d) show the shares of firms operating with the old and new technologies within each sector. Panel (e) and (f) show the profit rates of firms employing the old and new technologies within each sector. The uncoordinated implementation of the new technologies increases the profit rate only for those firms initially adopting the innovation, but the gradual diffusion of the new technologies results in lower levels of profitability for all firms over time. Panel (g) shows the real wage and the average rates of exploitation in both sectors. Because the nominal wage is constant and the new technology reduces the price of the consumption good, the real wage rises over time. Hence, the gains from technology reduce the rate of exploitation. Finally, panel (f) shows the wage and profit shares of value added for the entire economy. Corresponding to an increase in the real wage, the wage share also rises over time. The reduction in the price of production of the consumption good leads the real wage to rise faster than the productivity of labor, contributing to the fall in the average rate of profit.

9. Final Remarks

In this paper I developed an innovative evolutionary approach to integrate the principle of effective demand from Keynes, the macroeconomic aggregates from Kalecki, and the formation of long-run prices of production from Classical Political Economy. My approach combines the replicator dynamics from evolutionary Game Theory and Marx's model of a competitive economy with technical change. In an evolutionary setting the replicator dynamics offers a behavioral microfoundation for spontaneous and path-dependent interactions of multiple uncoordinated agents. The replicator equation allows for the formalization, in real time, of the feedback effects between decisions planned at the micro level and the unintended social outcomes at the macro level. Externalities are present in this setting as uncoordinated agents do not internalize the social consequences of their individual actions.

My framework demonstrates how aggregate demand determines the realization of values, the distribution of income between wages and profits, the equalization of profit rates, the diffusion of new techniques, and the convergence of market prices to prices of production. My evolutionary approach therefore offers a clearer and more precise presentation of Marx's system in *Capital III*. In particular, I bring together the formation of prices of production amid technological progress (as in chapter IX) and the role of aggregate demand in the realization of value (as in chapter XV).

Contrary to the Okishio theorem, which only holds under an exogenous real wage, technical change can lead profit rates to fall. Once the real wage is endogenous to aggregate demand, technical change increases the profit rate of early adopters. But competition and the diffusion of the new technique can reduce profitability over time if the repricing of means of production and consumption goods increases the real wage faster than the productivity of labor. As Basu (2019) demonstrated, because technical change impacts both the real wage and the productivity of labor, the trend of the profit rate derives from the relation between these two factors. Okishio's theorem only holds true if labor productivity rises faster than the real wage, and hence technological change causes the profit rate to rise over time. But if technical change causes the real wage to rise faster than labor productivity, the profit rate will fall. For the rate of exploitation to increase systematically over time the model would need to include equations for the labor market, allowing for the existence of involuntary unemployment and job insecurity. This extension of the model will be pursued in further work.

Keynes' principle of effective demand thus offers a better and more complete understanding of how aggregate demand determines the rates of exploitation, the rates of profit, the functional distribution of income, the diffusion of technological innovation, and the gravitation of market prices toward prices of production in a competitive economy.

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Appendix: Stability Analysis

In this Appendix I present the stability analysis of the long-run stationary state. To avoid unnecessary complications I suppose no technical change in either sector ($\upsilon_{i,t} = 0 \; or \; 1$) and that the updating share is 100% ($\mu = 1$).

Asymptotic stability means that the stationary state is both *stable* and an *attractor*, so the system converges to it over time (Scheinerman, 2000; Elaydi, 2005). In the one-dimensional replicator equation, asymptotic stability requires the payoff of a strategy to increase less than the competing payoff when the agents adopting that strategy increase their share in the population (Bowles 2006; Gintis 2009). The expected payoffs are the average profit rates within each sector; thus the stability condition is:

$$\frac{\mathrm{d}\bar{r}_{1,t}}{\mathrm{d}f_{1,t}} < \frac{\mathrm{d}\bar{r}_{2,t}}{\mathrm{d}f_{1,t}} \tag{A.1}$$

The model has a stationary state with equalized profit rates across sectors at $p_1^* = \frac{wl_1(1+r^*)}{1-a_{11}(1+r^*)}$; $p_2^* = \left[\frac{wl_1 a_{12}(1+r^*)}{1-a_{11}(1+r^*)} + wl_2\right](1+r^*)$; $\left(\frac{e_1}{e_2}\right)^* = \frac{\frac{p_1^*}{w}\cdot\frac{a_{11}}{l_1}+1}{\frac{p_1^*}{w}\cdot\frac{a_{12}}{l_2}+1}$; $\left(\frac{x_1}{x_2}\right)^* = \left(\frac{f_1^*}{1-f_1^*}\right)\frac{p_1^* a_{12}+wl_2}{p_1^* a_{11}+wl_1}$. With some algebraic manipulation, the stability condition (A.1) is satisfied when:

$$\gamma_1\left(\frac{f_{1,t-1}}{1+\tilde{r}_{t-1}}\right) - \gamma_2\left(\frac{f_{1,t-1}+\tilde{r}_{t-1}}{1+\tilde{r}_{t-1}}\right) < \left(\frac{f_{1,t}}{1-f_{1,t}}\right)^2\left(\frac{1}{\frac{p_{1,t-1}}{w}\frac{a_{11}}{l_1}+1}\right) \tag{A.2}$$

This inequality means that the stationary state $0 < f_1^* < 1$ tends to become less stable when, ceteris paribus: (i) the organic composition of capital in sector I $\left(\frac{p_{1,t-1}}{w}\frac{a_{11}}{l_1}\right)$ is very high; (ii) the ratio between the sector investment coefficients $\left(\frac{\gamma_1}{\gamma_2}\right)$ is very high. Computer simulations confirm these results and further indicate that the stationary state becomes less stable, ceteris paribus, at low values of the updating share μ.

But as long as γ_1 and γ_2 are not too far apart and the technical composition $\frac{a_{11}}{l_1}$ in sector I is not too high, the stationary state with equalized profit rates is asymptotically stable under the principle of effective demand.

The stability condition (A.1) refers to the one-dimensional replicator in which the economy has only two sectors, so the interior solution $0 < f_1^* < 1$ is either stable or unstable. In an economy with three or more sectors the stability condition in higher dimensions would cover cases with saddle path stability and limit cycles. This issue is beyond the scope of this paper but will be pursued in further work.

Foley (2018) and Cogliano (2011) have claimed that while competition between capital equalizes profit rates, competition between workers equalizes the rates of exploitation. To allow for the free flow of labor to arrive at $e_1^* = e_2^*$, the nominal wage in either one of the two sectors would need to vary such that $w_1 = \frac{\frac{a_{11}}{l_1}}{\frac{a_{12}}{l_2}} w_2$. But the adjustment of nominal wages across sectors according to this rule would hardly make any difference to the dynamics of the model.

Tables and Figures

Table 1: Stationary States and Asymptotic Properties

			Stationary States
(a) Macro Inter-Sector Replicator			
Sector I is ESS Sector II is ESS	$\Delta f_{i,t+1} = 0$	$\bar{r}_{1,t} \gtreqless \bar{r}_{2,t}$	$f_{1,t} = 1$ is stable $f_{1,t} = 0$ is stable $0 < f_{i,t}^{*} < 1$ is unstable
Sector I is ESS Sector II is not ESS	$\Delta f_{i,t+1} = 0$	$\bar{r}_{1,t} > \bar{r}_{2,t}$	$f_{1,t} = 1$ is stable $f_{1,t} = 0$ is unstable
Sector I is not ESS Sector II is ESS	$\Delta f_{i,t+1} = 0$	$\bar{r}_{1,t} < \bar{r}_{2,t}$	$f_{1,t} = 1$ is unstable $f_{1,t} = 0$ is stable
No ESS	$\Delta f_{i,t+1} = 0$	$\bar{r}_{1,t} = \bar{r}_{2,t}$	$0 < f_{i,t}^{*} < 1$ is stable $f_{1,t} = 1$ is unstable $f_{1,t} = 0$ is unstable
(b) Micro Intra-Sector Replicator in Sector I			
Innovate is ESS Not innovate is ESS	$\Delta v_{1,t+1} = 0$	$r_{1,t}^{n} \gtreqless r_{1,t}^{o}$	$v_{1,t} = 1$ is stable $v_{1,t} = 0$ is stable $0 < v_{1,t}^{*} < 1$ is unstable
Innovate is ESS Not innovate is not ESS	$\Delta v_{1,t+1} = 0$	$r_{1,t}^{n} > r_{1,t}^{o}$	$v_{1,t} = 1$ is stable $v_{1,t} = 0$ is unstable
Innovate is not ESS Not innovate is ESS	$\Delta v_{1,t+1} = 0$	$r_{1,t}^{n} < r_{1,t}^{o}$	$v_{1,t} = 1$ is unstable $v_{1,t} = 0$ is stable
No ESS	$\Delta v_{1,t+1} = 0$	$r_{1,t}^{n} = r_{1,t}^{o}$	$0 < v_{1,t}^{*} < 1$ is stable $v_{1,t} = 1$ is unstable $v_{1,t} = 0$ is unstable
(c) Micro Intra-Sector Replicator in Sector II			
Innovate is ESS Not innovate is ESS	$\Delta v_{2,t+1} = 0$	$r_{2,t}^{n} \gtreqless r_{2,t}^{o}$	$v_{2,t} = 1$ is stable $v_{2,t} = 0$ is stable $0 < v_{2,t}^{*} < 1$ is unstable
Innovate is ESS Not innovate is not ESS	$\Delta v_{2,t+1} = 0$	$r_{2,t}^{n} > r_{2,t}^{o}$	$v_{2,t} = 1$ is stable $v_{2,t} = 0$ is unstable
Innovate is not ESS Not innovate is ESS	$\Delta v_{2,t+1} = 0$	$r_{2,t}^{n} < r_{2,t}^{o}$	$v_{2,t} = 1$ is unstable $v_{2,t} = 0$ is stable
No ESS	$\Delta v_{2,t+1} = 0$	$r_{2,t}^{n} = r_{2,t}^{o}$	$0 < v_{2,t}^{*} < 1$ is stable $v_{2,t} = 1$ is unstable $v_{1,t} = 0$ is unstable

Figure 1: Simulation of the Evolutionary Model